

Question 10

Let $y = \ln(\sqrt{1 + e^{4x}})$.

- (a) Use the chain rule to find $\frac{dy}{dx}$.
- (b) Clearly indicate the nested functions and show each step of differentiation.

Question 10 - Solution

We are given:

$$y = \ln \left(\sqrt{1 + e^{4x}} \right)$$

First, simplify the expression:

$$y = \ln \left[(1 + e^{4x})^{1/2} \right] = \frac{1}{2} \ln(1 + e^{4x})$$

Now differentiate:

$$\frac{dy}{dx} = \frac{1}{2} \cdot \frac{1}{1 + e^{4x}} \cdot \frac{d}{dx}(1 + e^{4x})$$

Differentiate the inner exponential:

$$\frac{d}{dx}(1 + e^{4x}) = 4e^{4x}$$

Putting it all together:

$$\frac{dy}{dx} = \frac{1}{2} \cdot \frac{4e^{4x}}{1 + e^{4x}} = \boxed{\frac{2e^{4x}}{1 + e^{4x}}}$$

Nested Function Structure:

$$u = e^{4x}, v = 1 + u, w = \ln(\sqrt{v}) = \frac{1}{2} \ln(v)$$

Differentiation followed this path:

$$\frac{dw}{dv} \cdot \frac{dv}{du} \cdot \frac{du}{dx}$$