

Question 9

Let $f(x) = \sin^2(5x^3 + 1)$.

- (a) Use the chain rule to compute $f'(x)$.
- (b) Write down each layer of function composition clearly before differentiating.

Question 9 - Solution

We are given:

$$f(x) = \sin^2(5x^3 + 1)$$

This can be rewritten as:

$$f(x) = [\sin(5x^3 + 1)]^2$$

Layers of function composition:

1. Innermost function: $u(x) = 5x^3 + 1$ 2. Middle function: $v(u) = \sin(u)$ 3. Outer function: $f(v) = v^2$

Differentiate step by step:

Start from the outside:

$$f'(x) = 2 \cdot \sin(5x^3 + 1) \cdot \frac{d}{dx}[\sin(5x^3 + 1)]$$

Now differentiate the inner sine function:

$$\frac{d}{dx}[\sin(5x^3 + 1)] = \cos(5x^3 + 1) \cdot \frac{d}{dx}[5x^3 + 1] = \cos(5x^3 + 1) \cdot 15x^2$$

Putting it all together:

$$f'(x) = 2 \cdot \sin(5x^3 + 1) \cdot \cos(5x^3 + 1) \cdot 15x^2$$

$$\boxed{f'(x) = 30x^2 \cdot \sin(5x^3 + 1) \cdot \cos(5x^3 + 1)}$$

Alternatively, you may express the result using the identity:

$$2 \sin \theta \cos \theta = \sin(2\theta)$$

$$\Rightarrow f'(x) = 15x^2 \cdot \sin(2(5x^3 + 1)) = \boxed{15x^2 \cdot \sin(10x^3 + 2)}$$

Either boxed expression is valid.