

Question 8

Let $y = \ln\left(\sqrt{1 + e^{3x^2}}\right)$.

- (a) Use the chain rule to compute $\frac{dy}{dx}$.
- (b) Explain how the composition of functions influences the order of derivatives.

Question 8 - Solution

We are given:

$$y = \ln \left(\sqrt{1 + e^{3x^2}} \right)$$

We rewrite the square root as a power:

$$y = \ln \left((1 + e^{3x^2})^{1/2} \right)$$

Using the log rule:

$$y = \frac{1}{2} \ln(1 + e^{3x^2})$$

Differentiate:

$$\frac{dy}{dx} = \frac{1}{2} \cdot \frac{1}{1 + e^{3x^2}} \cdot \frac{d}{dx}[e^{3x^2}]$$

Differentiate the exponential:

$$\frac{d}{dx}[e^{3x^2}] = e^{3x^2} \cdot \frac{d}{dx}[3x^2] = e^{3x^2} \cdot 6x$$

Putting it all together:

$$\frac{dy}{dx} = \frac{1}{2} \cdot \frac{1}{1 + e^{3x^2}} \cdot e^{3x^2} \cdot 6x$$

Simplify:

$$\boxed{\frac{dy}{dx} = \frac{3xe^{3x^2}}{1 + e^{3x^2}}}$$

Explanation: This function has a nested composition: Innermost: $3x^2$, Exponential: e^{3x^2} , Added to 1, Square root $\rightarrow$ converted to a power, Then natural logarithm

At each step, the derivative is taken from the outside and multiplied by the derivative of the inside, following the chain rule.