

Question 7

Let $y = e^{\tan^3(2x)}$.

- (a) Use the chain rule to compute $\frac{dy}{dx}$.
- (b) Clearly outline the layers of composition and how each derivative is applied.

Question 7 - Solution

We are given:

$$y = e^{\tan^3(2x)} = e^{(\tan(2x))^3}$$

Let's identify the layers of composition:

- Outermost: e^u
- Middle: $u = v^3$
- Inner: $v = \tan(2x)$
- Innermost: $2x$

Apply the chain rule step-by-step:

$$\frac{dy}{dx} = \frac{d}{dx} \left(e^{(\tan(2x))^3} \right) = e^{(\tan(2x))^3} \cdot \frac{d}{dx} ((\tan(2x))^3)$$

Differentiate the power:

$$\frac{d}{dx} ((\tan(2x))^3) = 3(\tan(2x))^2 \cdot \frac{d}{dx} [\tan(2x)]$$

Differentiate $\tan(2x)$:

$$\frac{d}{dx} [\tan(2x)] = \sec^2(2x) \cdot 2$$

Now put it all together:

$$\frac{dy}{dx} = e^{(\tan(2x))^3} \cdot 3(\tan(2x))^2 \cdot \sec^2(2x) \cdot 2$$

Simplify:

$$\boxed{\frac{dy}{dx} = 6(\tan(2x))^2 \sec^2(2x) \cdot e^{(\tan(2x))^3}}$$