

Question 6

Let $f(x) = \ln\left(\sqrt{1 + \sin^2(5x)}\right)$.

- (a) Use the chain rule to compute $f'(x)$.
- (b) Identify the layers of composition involved and describe the differentiation process step-by-step.

Question 6 - Solution

We are given:

$$f(x) = \ln \left(\sqrt{1 + \sin^2(5x)} \right)$$

Let's simplify before differentiating:

$$f(x) = \ln \left((1 + \sin^2(5x))^{1/2} \right) = \frac{1}{2} \ln(1 + \sin^2(5x))$$

Now we differentiate:

$$f'(x) = \frac{1}{2} \cdot \frac{1}{1 + \sin^2(5x)} \cdot \frac{d}{dx}[\sin^2(5x)]$$

We apply the chain rule to $\sin^2(5x)$:

$$\frac{d}{dx}[\sin^2(5x)] = 2 \sin(5x) \cdot \cos(5x) \cdot 5 = 10 \sin(5x) \cos(5x)$$

Therefore:

$$f'(x) = \frac{1}{2} \cdot \frac{10 \sin(5x) \cos(5x)}{1 + \sin^2(5x)}$$

$$f'(x) = \frac{5 \sin(5x) \cos(5x)}{1 + \sin^2(5x)}$$