

Question 5

Let $f(x) = e^{\tan(x^3+1)}$.

- (a) Use the chain rule to find $f'(x)$.
- (b) Clearly identify each function used in the composition and the order in which the chain rule is applied.

Question 5 - Solution

We are given:

$$f(x) = e^{\tan(x^3+1)}$$

This is a composition of multiple functions. Let's break it down:

- Outer function: $u = e^v$
- Middle function: $v = \tan(w)$
- Inner function: $w = x^3 + 1$

Using the chain rule:

$$f'(x) = \frac{d}{dx} \left[e^{\tan(x^3+1)} \right] = e^{\tan(x^3+1)} \cdot \frac{d}{dx} [\tan(x^3 + 1)]$$

Now,

$$\frac{d}{dx} [\tan(x^3 + 1)] = \sec^2(x^3 + 1) \cdot \frac{d}{dx} [x^3 + 1] = \sec^2(x^3 + 1) \cdot 3x^2$$

Putting it all together:

$$f'(x) = e^{\tan(x^3+1)} \cdot \sec^2(x^3 + 1) \cdot 3x^2$$

$$\boxed{f'(x) = 3x^2 \cdot \sec^2(x^3 + 1) \cdot e^{\tan(x^3+1)}}$$