

Question 4

Let $y = \sqrt{1 + \sin^2(3x)}$.

- (a) Find $\frac{dy}{dx}$ using the chain rule.
- (b) Simplify the result as much as possible.

Question 4 - Solution

We are given:

$$y = \sqrt{1 + \sin^2(3x)} = (1 + \sin^2(3x))^{1/2}$$

Step 1: Apply the chain rule:

Let: - Outer function: $u^{1/2}$ - Inner function: $u = 1 + \sin^2(3x)$

We compute:

$$\frac{dy}{dx} = \frac{1}{2}(1 + \sin^2(3x))^{-1/2} \cdot \frac{d}{dx}[\sin^2(3x)]$$

Now differentiate $\sin^2(3x)$ using the chain rule:

$$\frac{d}{dx}[\sin^2(3x)] = 2 \sin(3x) \cdot \cos(3x) \cdot 3 = 6 \sin(3x) \cos(3x)$$

Step 2: Combine terms:

$$\frac{dy}{dx} = \frac{1}{2}(1 + \sin^2(3x))^{-1/2} \cdot 6 \sin(3x) \cos(3x)$$

$$\frac{dy}{dx} = \frac{3 \sin(3x) \cos(3x)}{\sqrt{1 + \sin^2(3x)}}$$

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