

Question 3

Let $y = \ln(\tan^3(5x^2 + 1))$.

- (a) Differentiate y with respect to x using the chain rule.
- (b) Simplify the derivative as much as possible.

Question 3 - Solution

We are given:

$$y = \ln(\tan^3(5x^2 + 1))$$

Step 1: Use logarithmic identity:

$$\ln(\tan^3(5x^2 + 1)) = 3 \ln(\tan(5x^2 + 1))$$

Step 2: Differentiate using chain rule:

$$\frac{dy}{dx} = 3 \cdot \frac{1}{\tan(5x^2 + 1)} \cdot \sec^2(5x^2 + 1) \cdot (10x)$$

Step 3: Simplify the expression:

$$\frac{dy}{dx} = \frac{30x \sec^2(5x^2 + 1)}{\tan(5x^2 + 1)}$$

$$\boxed{\frac{dy}{dx} = \frac{30x \sec^2(5x^2 + 1)}{\tan(5x^2 + 1)}}$$