

Question 2

Let $f(x) = \sin(\sqrt{1+x^4})$.

- (a) Find $f'(x)$ using the chain rule.
- (b) Evaluate $f'(1)$ exactly (do not approximate numerically).

Question 2 - Solution

We are given:

$$f(x) = \sin\left(\sqrt{1+x^4}\right)$$

Let's apply the chain rule step by step.

Step 1: Let

$$u(x) = \sqrt{1+x^4} = (1+x^4)^{1/2} \quad \text{so that} \quad f(x) = \sin(u(x))$$

Step 2: Apply the chain rule:

$$f'(x) = \cos(u(x)) \cdot u'(x)$$

Step 3: Differentiate $u(x)$:

$$u(x) = (1+x^4)^{1/2} \Rightarrow u'(x) = \frac{1}{2}(1+x^4)^{-1/2} \cdot 4x^3 = \frac{2x^3}{\sqrt{1+x^4}}$$

Final derivative:

$$f'(x) = \cos\left(\sqrt{1+x^4}\right) \cdot \frac{2x^3}{\sqrt{1+x^4}}$$

$$\boxed{f'(x) = \frac{2x^3 \cos\left(\sqrt{1+x^4}\right)}{\sqrt{1+x^4}}}$$

(b) Evaluate at $x = 1$:

$$f'(1) = \frac{2(1)^3 \cdot \cos\left(\sqrt{1+1^4}\right)}{\sqrt{1+1^4}} = \frac{2 \cdot \cos(\sqrt{2})}{\sqrt{2}}$$

$$\boxed{f'(1) = \frac{2 \cos(\sqrt{2})}{\sqrt{2}}}$$