

Question 1

Differentiate the function:

$$f(x) = \sqrt{1 + e^{3x^2}}$$

Clearly show how the chain rule is used in multiple layers.

Question 1 - Solution

We are given:

$$f(x) = \sqrt{1 + e^{3x^2}} = (1 + e^{3x^2})^{1/2}$$

Use the chain rule:

$$\begin{aligned} f'(x) &= \frac{1}{2}(1 + e^{3x^2})^{-1/2} \cdot \frac{d}{dx}(1 + e^{3x^2}) \\ &= \frac{1}{2\sqrt{1 + e^{3x^2}}} \cdot \frac{d}{dx}(e^{3x^2}) \end{aligned}$$

Now apply the chain rule again to e^{3x^2} :

$$\frac{d}{dx}(e^{3x^2}) = e^{3x^2} \cdot \frac{d}{dx}(3x^2) = e^{3x^2} \cdot 6x$$

Putting it all together:

$$f'(x) = \frac{1}{2\sqrt{1 + e^{3x^2}}} \cdot e^{3x^2} \cdot 6x = \boxed{\frac{3x \cdot e^{3x^2}}{\sqrt{1 + e^{3x^2}}}}$$