

**Question 10**

Let  $f(x) = \frac{\sinh(3x)}{\cosh(x)}$ .

- (a) Differentiate  $f(x)$  using the quotient rule and known derivatives of hyperbolic functions.
- (b) Simplify the expression for  $f'(x)$  as much as possible.

**Question 10 - Solution**

We are given:

$$f(x) = \frac{\sinh(3x)}{\cosh(x)}$$

**(a) Apply the quotient rule:**

The quotient rule is:

$$\left(\frac{u}{v}\right)' = \frac{u'v - uv'}{v^2}$$

Let:

$$u = \sinh(3x), \quad v = \cosh(x)$$

Differentiate:

$$u' = 3 \cosh(3x), \quad v' = \sinh(x)$$

Apply the rule:

$$f'(x) = \frac{3 \cosh(3x) \cosh(x) - \sinh(3x) \sinh(x)}{\cosh^2(x)}$$

**(b) Final simplified expression:**

|                                                                        |
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| $f'(x) = \frac{3 \cosh(3x) \cosh(x) - \sinh(3x) \sinh(x)}{\cosh^2(x)}$ |
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