

Question 9

Let $f(x) = \tanh(x^2 + 1)$.

- (a) Find the derivative $f'(x)$ using the chain rule and hyperbolic identities.
- (b) Determine the value of the derivative at $x = 0$.

Question 9 - Solution

We are given:

$$f(x) = \tanh(x^2 + 1)$$

(a) Find $f'(x)$:

Use the chain rule:

$$f'(x) = \frac{d}{dx} [\tanh(u)] \cdot \frac{d}{dx} (x^2 + 1)$$

Recall:

$$\frac{d}{dx} [\tanh(u)] = \operatorname{sech}^2(u)$$

So let $u = x^2 + 1$, then:

$$f'(x) = \operatorname{sech}^2(x^2 + 1) \cdot 2x$$

$$\boxed{f'(x) = 2x \cdot \operatorname{sech}^2(x^2 + 1)}$$

(b) Evaluate at $x = 0$:

$$f'(0) = 2 \cdot 0 \cdot \operatorname{sech}^2(0^2 + 1) = 0$$

$$\boxed{f'(0) = 0}$$