

Question 8

Let $f(x) = \frac{\sinh(3x)}{x^2+1}$.

- (a) Find the derivative $f'(x)$ using the quotient rule.
- (b) Evaluate the derivative at $x = 0$.

Question 8 - Solution

We are given:

$$f(x) = \frac{\sinh(3x)}{x^2 + 1}$$

(a) Find $f'(x)$:

This is a quotient, so we use the quotient rule:

$$f'(x) = \frac{g'(x)h(x) - g(x)h'(x)}{[h(x)]^2}$$

Let:

$$g(x) = \sinh(3x), \quad h(x) = x^2 + 1$$

Then:

$$g'(x) = 3 \cosh(3x), \quad h'(x) = 2x$$

So:

$$f'(x) = \frac{3 \cosh(3x)(x^2 + 1) - \sinh(3x)(2x)}{(x^2 + 1)^2}$$

$$f'(x) = \frac{3(x^2 + 1) \cosh(3x) - 2x \sinh(3x)}{(x^2 + 1)^2}$$

(b) Evaluate at $x = 0$:

First note:

$$\cosh(0) = 1, \quad \sinh(0) = 0$$

Then:

$$f'(0) = \frac{3(0^2 + 1) \cdot 1 - 0}{(0^2 + 1)^2} = \frac{3}{1} = 3$$

$$f'(0) = 3$$