

Question 7

Let $f(x) = \tanh(x^2) \cdot \operatorname{sech}(x)$.

- (a) Differentiate $f(x)$ using the product rule and the chain rule.
- (b) Simplify the derivative as much as possible using hyperbolic identities.

Question 7 - Solution

We are given:

$$f(x) = \tanh(x^2) \cdot \operatorname{sech}(x)$$

(a) Use the product rule:

Let:

$$u = \tanh(x^2), \quad v = \operatorname{sech}(x)$$

Then:

$$f'(x) = u'v + uv'$$

Differentiate each part:

Step 1: $u = \tanh(x^2) \Rightarrow u' = \operatorname{sech}^2(x^2) \cdot 2x$ using chain rule

Step 2: $v = \operatorname{sech}(x) \Rightarrow v' = -\operatorname{sech}(x) \cdot \tanh(x)$

Now substitute into the product rule:

$$f'(x) = [\operatorname{sech}^2(x^2) \cdot 2x] \cdot \operatorname{sech}(x) + \tanh(x^2) \cdot [-\operatorname{sech}(x) \cdot \tanh(x)]$$

(b) Final expression:

$$f'(x) = 2x \operatorname{sech}^2(x^2) \operatorname{sech}(x) - \tanh(x^2) \operatorname{sech}(x) \tanh(x)$$