

Question 6

Let $f(x) = x \cdot \cosh(x^2)$.

- (a) Find $f'(x)$.
- (b) Determine the value of the derivative at $x = 0$.
- (c) Is there a local minimum, maximum, or neither at $x = 0$? Justify using the derivative.

Question 6 - Solution

We are given:

$$f(x) = x \cdot \cosh(x^2)$$

(a) Find $f'(x)$:

This is a product of two functions: Let $u = x$, $v = \cosh(x^2)$

Then:

$$f'(x) = u'v + uv' = 1 \cdot \cosh(x^2) + x \cdot \frac{d}{dx}[\cosh(x^2)]$$

Differentiate $\cosh(x^2)$ using the chain rule:

$$\frac{d}{dx}[\cosh(x^2)] = \sinh(x^2) \cdot 2x$$

So:

$$f'(x) = \cosh(x^2) + x \cdot (2x \cdot \sinh(x^2)) = \cosh(x^2) + 2x^2 \sinh(x^2)$$

$$\boxed{f'(x) = \cosh(x^2) + 2x^2 \sinh(x^2)}$$

(b) Value at $x = 0$:

$$f'(0) = \cosh(0^2) + 2 \cdot 0^2 \cdot \sinh(0^2) = \cosh(0) = 1$$

$$\boxed{f'(0) = 1}$$

(c) Local Behavior at $x = 0$:

Since $f'(0) = 1 > 0$, the function is increasing at $x = 0$, so:

Conclusion: There is **no local minimum or maximum** at $x = 0$, because the derivative is positive and the function is increasing there.