

Question 5

Let $f(x) = \frac{\sinh(x)}{\cosh(x)}$.

- (a) Simplify the function if possible.
- (b) Find $f'(x)$.
- (c) Determine the intervals where $f(x)$ is increasing or decreasing.

Question 5 - Solution**(a) Simplify the Function:**

We have:

$$f(x) = \frac{\sinh(x)}{\cosh(x)}$$

This is the definition of the hyperbolic tangent function:

$$f(x) = \tanh(x)$$

(b) Differentiate $f(x) = \tanh(x)$:

The derivative of $\tanh(x)$ is:

$$f'(x) = \operatorname{sech}^2(x)$$

(c) Increasing/Decreasing Intervals:

Since $f'(x) = \operatorname{sech}^2(x) > 0$ for all real x , the function is strictly increasing on its entire domain.

Conclusion:

$f'(x) = \operatorname{sech}^2(x), \quad f(x) \text{ is increasing on } (-\infty, \infty)$
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