

Question 4

Let $f(x) = \frac{\cosh^2(x)-1}{\sinh(x)}$.

- (a) Simplify the expression before differentiating.
- (b) Compute $f'(x)$.
- (c) Identify any points where $f'(x)$ is undefined.

Question 4 - Solution**(a) Simplification:**

We start with:

$$f(x) = \frac{\cosh^2(x) - 1}{\sinh(x)}$$

Use the identity:

$$\cosh^2(x) - 1 = \sinh^2(x)$$

Thus:

$$f(x) = \frac{\sinh^2(x)}{\sinh(x)} = \sinh(x)$$

(b) Differentiate:

$$f'(x) = \frac{d}{dx} \sinh(x) = \cosh(x)$$

(c) Points of Non-Differentiability:

Since $f(x) = \sinh(x)$ is differentiable for all real x , and the simplification is valid as long as the denominator of the original function $\sinh(x) \neq 0$, we must consider the domain of the original function.

The original function is undefined at:

$$\sinh(x) = 0 \Rightarrow x = 0$$

So although $f(x) = \sinh(x)$ is differentiable everywhere, the original form $f(x) = \frac{\cosh^2(x)-1}{\sinh(x)}$ is undefined at $x = 0$, and thus f is not defined or differentiable there in its original form.

Conclusion:

$f'(x) = \cosh(x), \quad f'(x) \text{ undefined at } x = 0$
