

Question 3

Let $f(x) = \tanh(x) \sinh(x)$.

- (a) Compute $f'(x)$.
- (b) Simplify your result as much as possible using hyperbolic identities.

Question 3 - Solution

We are given:

$$f(x) = \tanh(x) \sinh(x)$$

This is a product of two functions, so we apply the product rule:

$$f'(x) = \tanh'(x) \sinh(x) + \tanh(x) \sinh'(x)$$

Recall the derivatives:

$$\tanh'(x) = \operatorname{sech}^2(x), \quad \sinh'(x) = \cosh(x)$$

Substitute:

$$f'(x) = \operatorname{sech}^2(x) \sinh(x) + \tanh(x) \cosh(x)$$

Now simplify. Recall:

$$\tanh(x) = \frac{\sinh(x)}{\cosh(x)} \Rightarrow \tanh(x) \cosh(x) = \sinh(x)$$

So:

$$f'(x) = \operatorname{sech}^2(x) \sinh(x) + \sinh(x) = \sinh(x) (\operatorname{sech}^2(x) + 1)$$

Final Answer:

$$\boxed{f'(x) = \sinh(x) (\operatorname{sech}^2(x) + 1)}$$