

Question 2

Let $f(x) = \frac{\sinh(3x)}{\cosh(x)}$.

- (a) Compute $f'(x)$ using differentiation rules.
- (b) Simplify the result as much as possible using hyperbolic identities.

Question 2 - Solution

The quotient rule gives

$$f'(x) = \frac{3 \cosh(3x) \cosh x - \sinh(3x) \sinh x}{\cosh^2 x}.$$

Use the product identities

$$\cosh(3x) \cosh x = \frac{1}{2}(\cosh 4x + \cosh 2x),$$

$$\sinh(3x) \sinh x = \frac{1}{2}(\cosh 4x - \cosh 2x).$$

Thus

$$f'(x) = \frac{\cosh 4x + 2 \cosh 2x}{\cosh^2 x}.$$

At zero the derivative is 3, agreeing with the original quotient-rule expression.