

Question 10

Let $f(x) = \tan^{-1}(\sqrt{1-x^2})$ for $-1 < x < 1$.

- (a) Find $f'(x)$.
- (b) Simplify your answer as much as possible.

Question 10 - Solution

We are given:

$$f(x) = \tan^{-1}(\sqrt{1-x^2})$$

Let $u = \sqrt{1-x^2}$, then by the chain rule:

$$f'(x) = \frac{1}{1+u^2} \cdot u'$$

Step 1: Compute u' :

$$u = (1-x^2)^{1/2} \Rightarrow u' = \frac{1}{2}(1-x^2)^{-1/2} \cdot (-2x) = \frac{-x}{\sqrt{1-x^2}}$$

Step 2: Compute $1+u^2$:

$$1+u^2 = 1+(1-x^2) = 2-x^2$$

Step 3: Combine everything:

$$f'(x) = \frac{1}{1+u^2} \cdot u' = \frac{1}{2-x^2} \cdot \frac{-x}{\sqrt{1-x^2}} = \boxed{\frac{-x}{(2-x^2)\sqrt{1-x^2}}}$$