

Question 8

Let $f(x) = \tan^{-1}(\sqrt{1-x^2})$, where $-1 < x < 1$.

- (a) Compute $f'(x)$.
- (b) Simplify your result as much as possible.

Question 8 - Solution

We are given:

$$f(x) = \tan^{-1}(\sqrt{1-x^2})$$

(a) Differentiate using the chain rule:

Let $u = \sqrt{1-x^2}$, so:

$$f(x) = \tan^{-1}(u) \Rightarrow f'(x) = \frac{1}{1+u^2} \cdot \frac{du}{dx}$$

Now compute each part:

$$u = (1-x^2)^{1/2}, \quad \frac{du}{dx} = \frac{1}{2}(1-x^2)^{-1/2} \cdot (-2x) = \frac{-x}{\sqrt{1-x^2}}$$

Now plug into the derivative:

$$f'(x) = \frac{1}{1+(1-x^2)} \cdot \frac{-x}{\sqrt{1-x^2}} = \frac{1}{2-x^2} \cdot \frac{-x}{\sqrt{1-x^2}}$$

$$f'(x) = \boxed{\frac{-x}{(2-x^2)\sqrt{1-x^2}}}$$

(b) Final simplified form:

$$\boxed{f'(x) = \frac{-x}{(2-x^2)\sqrt{1-x^2}}}$$