

Question 6

Let

$$f(x) = \tan^{-1} \left(\sqrt{1 - x^2} \right)$$

- (a) Find the derivative $f'(x)$.
- (b) Determine the domain of $f'(x)$.

Question 6 - Solution

We are given:

$$f(x) = \tan^{-1}(\sqrt{1-x^2})$$

(a) Finding $f'(x)$:

Let $u(x) = \sqrt{1-x^2}$, so:

$$f(x) = \tan^{-1}(u) \Rightarrow f'(x) = \frac{1}{1+u^2} \cdot u'(x)$$

First, compute $u'(x)$:

$$u(x) = (1-x^2)^{1/2} \Rightarrow u'(x) = \frac{1}{2}(1-x^2)^{-1/2} \cdot (-2x) = \frac{-x}{\sqrt{1-x^2}}$$

Now compute:

$$f'(x) = \frac{1}{1+(1-x^2)} \cdot \frac{-x}{\sqrt{1-x^2}} = \frac{1}{2-x^2} \cdot \frac{-x}{\sqrt{1-x^2}}$$

Final Answer:

$$f'(x) = \frac{-x}{(2-x^2)\sqrt{1-x^2}}$$

(b) Domain of $f'(x)$:

We need: $\sqrt{1-x^2}$ to be real $1-x^2 \geq 0 \Rightarrow x \in [-1, 1]$, Denominator $\sqrt{1-x^2} \neq 0$ exclude $x = \pm 1$,
Also ensure $2-x^2 \neq 0 \Rightarrow x \neq \sqrt{2}, -\sqrt{2}$ (but these are already excluded)

So domain of $f'(x)$ is:

$$(-1, 1)$$