

Question 4

Let

$$f(x) = \tan^{-1}(\sqrt{1-x^2})$$

- (a) Determine the domain of $f(x)$.
- (b) Compute $f'(x)$ for $x \in (-1, 1)$.

Question 4 - Solution**(a) Domain:**

Since $f(x) = \tan^{-1}(\sqrt{1-x^2})$, we require the expression under the square root to be non-negative:

$$1 - x^2 \geq 0 \Rightarrow x^2 \leq 1 \Rightarrow -1 \leq x \leq 1$$

But the square root function $\sqrt{1-x^2}$ is only real for $x \in [-1, 1]$, and since it appears inside an inverse tangent (which is defined for all real inputs), the domain of f is:

$$\boxed{[-1, 1]}$$

(b) Compute the derivative:

Let's differentiate using the chain rule:

$$f(x) = \tan^{-1}(u), \quad \text{where } u = \sqrt{1-x^2} \Rightarrow f'(x) = \frac{1}{1+u^2} \cdot \frac{du}{dx}$$

Now compute $\frac{du}{dx}$ where $u = \sqrt{1-x^2}$:

$$\frac{du}{dx} = \frac{1}{2\sqrt{1-x^2}} \cdot (-2x) = \frac{-x}{\sqrt{1-x^2}}$$

Next, compute $1+u^2 = 1+(1-x^2) = 2-x^2$

So:

$$f'(x) = \frac{1}{2-x^2} \cdot \left(\frac{-x}{\sqrt{1-x^2}} \right) = \boxed{\frac{-x}{(2-x^2)\sqrt{1-x^2}}}$$