

Question 1

Differentiate the function:

$$f(x) = x \cdot \arcsin(x)$$

Then evaluate $f'(x)$ at $x = \frac{1}{2}$.

Question 1 - Solution

We are given:

$$f(x) = x \cdot \arcsin(x)$$

Use the product rule:

$$f'(x) = \arcsin(x) + x \cdot \frac{1}{\sqrt{1-x^2}}$$

Answer:

$$f'(x) = \boxed{\arcsin(x) + \frac{x}{\sqrt{1-x^2}}}$$

Now evaluate at $x = \frac{1}{2}$:

$$f'\left(\frac{1}{2}\right) = \arcsin\left(\frac{1}{2}\right) + \frac{\frac{1}{2}}{\sqrt{1-\left(\frac{1}{2}\right)^2}} = \frac{\pi}{6} + \frac{1/2}{\sqrt{3}/2} = \frac{\pi}{6} + \frac{1}{\sqrt{3}}$$

Answer: $f'\left(\frac{1}{2}\right) = \boxed{\frac{\pi}{6} + \frac{1}{\sqrt{3}}}$