

Question 8

Let $f(x) = x^x$, where $x > 0$.

- (a) Find the derivative $f'(x)$.
- (b) Simplify your answer as much as possible.

Question 8 - Solution

We are given:

$$f(x) = x^x \quad \text{for } x > 0$$

This is a function where the variable is both the base and the exponent. To differentiate it, we take the natural logarithm of both sides:

Let

$$y = x^x$$

Take the natural log of both sides:

$$\ln y = \ln(x^x) = x \ln x$$

Differentiate both sides implicitly:

$$\frac{1}{y} \cdot \frac{dy}{dx} = \frac{d}{dx}(x \ln x)$$

Now differentiate the right-hand side using the product rule:

$$\frac{d}{dx}(x \ln x) = 1 \cdot \ln x + x \cdot \frac{1}{x} = \ln x + 1$$

Now solve for $\frac{dy}{dx}$:

$$\frac{dy}{dx} = y(\ln x + 1)$$

Recall that $y = x^x$, so:

$$f'(x) = \boxed{x^x(\ln x + 1)}$$