

**Question 7**

Let  $f(x) = \ln\left(\frac{e^x+1}{e^x-1}\right)$ , for  $x > 0$ .

- (a) Find  $f'(x)$ .
- (b) Simplify your answer as much as possible.

**Question 7 - Solution**

We are given:

$$f(x) = \ln \left( \frac{e^x + 1}{e^x - 1} \right)$$

**Step 1: Use the derivative of the natural log of a quotient:**

$$\frac{d}{dx} \ln \left( \frac{u(x)}{v(x)} \right) = \frac{d}{dx} [\ln(u(x)) - \ln(v(x))] = \frac{u'(x)}{u(x)} - \frac{v'(x)}{v(x)}$$

Here,  $u(x) = e^x + 1$ , and  $v(x) = e^x - 1$

So,

$$f'(x) = \frac{d}{dx} [\ln(e^x + 1) - \ln(e^x - 1)] = \frac{e^x}{e^x + 1} - \frac{e^x}{e^x - 1}$$

**Step 2: Combine the two terms:**

$$f'(x) = \frac{e^x}{e^x + 1} - \frac{e^x}{e^x - 1}$$

Get a common denominator:

$$f'(x) = e^x \left( \frac{1}{e^x + 1} - \frac{1}{e^x - 1} \right) = e^x \left( \frac{(e^x - 1) - (e^x + 1)}{(e^x + 1)(e^x - 1)} \right) = e^x \left( \frac{e^x - 1 - e^x - 1}{e^{2x} - 1} \right)$$

$$f'(x) = e^x \left( \frac{-2}{e^{2x} - 1} \right) = \boxed{\frac{-2e^x}{e^{2x} - 1}}$$