

Question 6

Let $f(x) = x^x$, defined for $x > 0$.

- (a) Find the derivative $f'(x)$.
- (b) Evaluate $f'(1)$.

Question 6 - Solution

We are given:

$$f(x) = x^x, \quad x > 0$$

This is a function where the base and exponent both depend on x . To differentiate it, we take logarithms on both sides.

Step 1: Take natural log of both sides:

Let $y = x^x$. Then,

$$\ln y = \ln(x^x) = x \ln x$$

Step 2: Differentiate both sides implicitly:

$$\frac{1}{y} \cdot \frac{dy}{dx} = \frac{d}{dx}(x \ln x) = \ln x + 1$$

Step 3: Solve for $\frac{dy}{dx}$:

$$\frac{dy}{dx} = y(\ln x + 1) = x^x(\ln x + 1)$$

Final answer:

$$\boxed{f'(x) = x^x(\ln x + 1)}$$

(b) Evaluate $f'(1)$:

$$f'(1) = 1^1(\ln 1 + 1) = 1(0 + 1) = \boxed{1}$$