

Question 5

Let the function $f(x) = \ln\left(\frac{x^2\sqrt{e^x+1}}{\ln x}\right)$, for $x > 1$.

- (a) Simplify the expression using logarithmic properties.
- (b) Find the derivative $f'(x)$.

Question 5 - Solution

We are given:

$$f(x) = \ln \left(\frac{x^2 \sqrt{e^x + 1}}{\ln x} \right), \quad x > 1$$

(a) Simplify the expression:

Use logarithmic identities:

$$\ln \left(\frac{A \cdot B}{C} \right) = \ln(A) + \ln(B) - \ln(C)$$

Apply to this expression:

$$f(x) = \ln(x^2) + \ln \left((e^x + 1)^{1/2} \right) - \ln(\ln x)$$

Now simplify powers:

$$f(x) = 2 \ln x + \frac{1}{2} \ln(e^x + 1) - \ln(\ln x)$$

(b) Differentiate $f(x)$:

Differentiate term-by-term:

$$\begin{aligned} \frac{d}{dx} [2 \ln x] &= \frac{2}{x} \\ \frac{d}{dx} \left[\frac{1}{2} \ln(e^x + 1) \right] &= \frac{1}{2} \cdot \frac{e^x}{e^x + 1} \\ \frac{d}{dx} [\ln(\ln x)] &= \frac{1}{x \ln x} \end{aligned}$$

Putting it all together:

$$f'(x) = \frac{2}{x} + \frac{1}{2} \cdot \frac{e^x}{e^x + 1} - \frac{1}{x \ln x}$$

Final Answer:

$f'(x) = \frac{2}{x} + \frac{e^x}{2(e^x + 1)} - \frac{1}{x \ln x}$
--