

Question 3

Let $f(x) = x^x$ for $x > 0$.

- (a) Use logarithmic differentiation to find $f'(x)$.
- (b) Evaluate $f'(x)$ at $x = 2$.

Question 3 - Solution

We are given:

$$f(x) = x^x, \quad x > 0$$

(a) Logarithmic Differentiation

Take natural log on both sides:

$$\ln(f(x)) = \ln(x^x) = x \ln(x)$$

Differentiate both sides implicitly:

$$\frac{1}{f(x)} \cdot f'(x) = \frac{d}{dx}[x \ln(x)]$$

Differentiate the right-hand side using product rule:

$$\frac{d}{dx}[x \ln(x)] = \ln(x) + 1$$

So:

$$\frac{1}{f(x)} \cdot f'(x) = \ln(x) + 1 \Rightarrow f'(x) = f(x) \cdot (\ln(x) + 1)$$

Recall $f(x) = x^x$, so:

$$\boxed{f'(x) = x^x(\ln(x) + 1)}$$

(b) Evaluate at $x = 2$:

$$f'(2) = 2^2(\ln(2) + 1) = 4(\ln(2) + 1)$$

Exact Answer:

$$\boxed{f'(2) = 4(\ln(2) + 1)}$$