

Question 2

Consider the function:

$$f(x) = \frac{e^x}{\ln(x)}$$

- (a) Find the derivative $f'(x)$.
- (b) State the domain of $f(x)$ and $f'(x)$.

Question 2 - Solution

We are given:

$$f(x) = \frac{e^x}{\ln(x)}$$

(a) Differentiate $f(x)$:

This is a quotient, so we apply the quotient rule:

$$f'(x) = \frac{(\ln(x) \cdot \frac{d}{dx}[e^x]) - (e^x \cdot \frac{d}{dx}[\ln(x)])}{(\ln(x))^2}$$

Differentiate:

$$\frac{d}{dx}[e^x] = e^x, \quad \frac{d}{dx}[\ln(x)] = \frac{1}{x}$$

Now substitute:

$$f'(x) = \frac{\ln(x) \cdot e^x - e^x \cdot \frac{1}{x}}{(\ln(x))^2} = \frac{e^x (\ln(x) - \frac{1}{x})}{(\ln(x))^2}$$

$$f'(x) = \frac{e^x (\ln(x) - \frac{1}{x})}{(\ln(x))^2}$$

(b) Domain of $f(x)$:

$\ln(x)$ is only defined for $x > 0$

But we also need $\ln(x) \neq 0 \Rightarrow x \neq 1$

So domain of $f(x)$ is:

$$(0, 1) \cup (1, \infty)$$

The derivative involves the same terms (no new restrictions), so:

$$\text{Domain of } f'(x) = \boxed{(0, 1) \cup (1, \infty)}$$