

Question 10

Consider the function:

$$f(x) = \sin(x) \cos(x)$$

- (a) Differentiate $f(x)$ using the product rule.
- (b) Simplify the derivative using trigonometric identities.
- (c) Evaluate $f'(x)$ at $x = \frac{\pi}{3}$.

Question 10 - Solution

We are given:

$$f(x) = \sin(x) \cos(x)$$

(a) Differentiate Using the Product Rule:

Let:

$$u(x) = \sin(x), \quad v(x) = \cos(x)$$

Then:

$$u'(x) = \cos(x), \quad v'(x) = -\sin(x)$$

Using the product rule:

$$f'(x) = u'(x)v(x) + u(x)v'(x) = \cos(x)\cos(x) + \sin(x)(-\sin(x)) = \cos^2(x) - \sin^2(x)$$

$$\boxed{f'(x) = \cos^2(x) - \sin^2(x)}$$

(b) Simplify Using Trig Identity:

We use the identity:

$$\cos^2(x) - \sin^2(x) = \cos(2x)$$

So:

$$\boxed{f'(x) = \cos(2x)}$$

(c) Evaluate at $x = \frac{\pi}{3}$:

$$f'\left(\frac{\pi}{3}\right) = \cos\left(2 \cdot \frac{\pi}{3}\right) = \cos\left(\frac{2\pi}{3}\right) = -\frac{1}{2}$$

$$\boxed{f'\left(\frac{\pi}{3}\right) = -\frac{1}{2}}$$