

Question 9

Let

$$f(x) = x \tan(x)$$

- (a) Use the product rule to find the derivative of $f(x)$.
- (b) Find the equation of the tangent line to $f(x)$ at $x = \frac{\pi}{4}$.

Question 9 - Solution

We are given:

$$f(x) = x \tan(x)$$

(a) Differentiate Using the Product Rule:

Let:

$$u(x) = x, \quad v(x) = \tan(x)$$

Then:

$$u'(x) = 1, \quad v'(x) = \sec^2(x)$$

Product rule:

$$f'(x) = u'(x)v(x) + u(x)v'(x) = 1 \cdot \tan(x) + x \cdot \sec^2(x) = \tan(x) + x \sec^2(x)$$

$$\boxed{f'(x) = \tan(x) + x \sec^2(x)}$$

(b) Tangent Line at $x = \frac{\pi}{4}$:

Evaluate:

$$f\left(\frac{\pi}{4}\right) = \frac{\pi}{4} \cdot \tan\left(\frac{\pi}{4}\right) = \frac{\pi}{4} \cdot 1 = \frac{\pi}{4}$$

$$f'\left(\frac{\pi}{4}\right) = \tan\left(\frac{\pi}{4}\right) + \frac{\pi}{4} \cdot \sec^2\left(\frac{\pi}{4}\right) = 1 + \frac{\pi}{4} \cdot 2 = 1 + \frac{\pi}{2}$$

Use point-slope form:

$$y - f\left(\frac{\pi}{4}\right) = f'\left(\frac{\pi}{4}\right) \left(x - \frac{\pi}{4}\right) \Rightarrow y - \frac{\pi}{4} = \left(1 + \frac{\pi}{2}\right) \left(x - \frac{\pi}{4}\right)$$

Final Answer: The tangent line is:

$$\boxed{y = \left(1 + \frac{\pi}{2}\right) \left(x - \frac{\pi}{4}\right) + \frac{\pi}{4}}$$