

Question 7

Let

$$f(x) = \tan(x) \cdot \sec(x)$$

- (a) Find the derivative $f'(x)$.
- (b) Simplify the result using trigonometric identities.

Question 7 - Solution

We are given:

$$f(x) = \tan(x) \cdot \sec(x)$$

(a) Apply the Product Rule:

The product rule states:

$$f'(x) = u'(x)v(x) + u(x)v'(x)$$

Let:

$$u(x) = \tan(x), \quad v(x) = \sec(x)$$

$$u'(x) = \sec^2(x), \quad v'(x) = \sec(x) \tan(x)$$

Then:

$$\begin{aligned} f'(x) &= \sec^2(x) \cdot \sec(x) + \tan(x) \cdot \sec(x) \cdot \tan(x) \\ &= \sec^3(x) + \sec(x) \tan^2(x) \end{aligned}$$

(b) Factor the result:

Both terms have a factor of $\sec(x)$, so factor it out:

$$f'(x) = \sec(x) (\sec^2(x) + \tan^2(x))$$

Final Answer:

$$\boxed{f'(x) = \sec(x) (\sec^2(x) + \tan^2(x))}$$