

Question 6

Let

$$f(x) = \frac{\sin(x)}{1 + \cos(x)}$$

- (a) Find the derivative $f'(x)$.
- (b) Simplify the derivative as much as possible.

Question 6 - Solution

We are given:

$$f(x) = \frac{\sin(x)}{1 + \cos(x)}$$

(a) Use the Quotient Rule:

The quotient rule is:

$$f'(x) = \frac{g'(x)h(x) - g(x)h'(x)}{[h(x)]^2}$$

Let:

$$g(x) = \sin(x), \quad h(x) = 1 + \cos(x)$$

$$g'(x) = \cos(x), \quad h'(x) = -\sin(x)$$

Then:

$$\begin{aligned} f'(x) &= \frac{\cos(x)(1 + \cos(x)) - \sin(x)(-\sin(x))}{(1 + \cos(x))^2} \\ &= \frac{\cos(x)(1 + \cos(x)) + \sin^2(x)}{(1 + \cos(x))^2} \end{aligned}$$

(b) Simplify the numerator:

First expand:

$$\cos(x)(1 + \cos(x)) = \cos(x) + \cos^2(x)$$

So the full numerator is:

$$\cos(x) + \cos^2(x) + \sin^2(x)$$

Use the identity:

$$\cos^2(x) + \sin^2(x) = 1$$

So the numerator becomes:

$$\cos(x) + 1$$

Therefore,

$$f'(x) = \frac{\cos(x) + 1}{(1 + \cos(x))^2}$$

Simplify:

$$f'(x) = \frac{1}{1 + \cos(x)}$$

Final Answer:

$$\boxed{f'(x) = \frac{1}{1 + \cos(x)}}$$