

Question 5

Let

$$f(x) = x \tan(x) - \sec(x)$$

- (a) Find the derivative $f'(x)$.
- (b) Identify any values of x where $f'(x)$ is undefined.

Question 5 - Solution

We are given:

$$f(x) = x \tan(x) - \sec(x)$$

(a) Differentiate each term.

Term 1: $x \tan(x)$ Use the product rule:

$$\frac{d}{dx}[x \tan(x)] = x \cdot \sec^2(x) + \tan(x)$$

Term 2: $-\sec(x)$

$$\frac{d}{dx}[-\sec(x)] = -\sec(x) \tan(x)$$

Combine the terms:

$$f'(x) = x \sec^2(x) + \tan(x) - \sec(x) \tan(x)$$

Final Answer:

$$f'(x) = x \sec^2(x) + \tan(x) - \sec(x) \tan(x)$$

(b) Values where $f'(x)$ is undefined:

Note that $\tan(x)$ and $\sec(x)$ are undefined at:

$$x = \frac{\pi}{2} + n\pi, \quad n \in \mathbb{Z}$$

So, the derivative $f'(x)$ is undefined at:

$$x = \frac{\pi}{2} + n\pi, \quad n \in \mathbb{Z}$$