

Question 4

Let

$$f(x) = \cot(x) \csc(x) + \cos^2(x)$$

- (a) Find the derivative $f'(x)$.
- (b) Simplify the expression as much as possible.

Question 4 - Solution

We are given:

$$f(x) = \cot(x) \csc(x) + \cos^2(x)$$

(a) Differentiate each term separately.

Term 1: $\cot(x) \csc(x)$

Using the product rule:

$$\begin{aligned}\frac{d}{dx}[\cot(x) \csc(x)] &= \cot(x)(-\csc(x)) + \csc(x)(-\csc^2(x)) \\ &= -\cot^2(x) \csc(x) - \csc^3(x)\end{aligned}$$

Term 2: $\cos^2(x)$

Using the chain rule:

$$\frac{d}{dx}[\cos^2(x)] = 2 \cos(x)(-\sin(x)) = -2 \sin(x) \cos(x)$$

(b) Combine and simplify:

$$f'(x) = -\cot^2(x) \csc(x) - \csc^3(x) - 2 \sin(x) \cos(x)$$

Use the identities:

$$\cot^2(x) = \csc^2(x) - 1 \quad 2 \sin(x) \cos(x) = \sin(2x)$$

$$-\cot^2(x) \csc(x) = -(\csc^2(x) - 1) \csc(x) = -\csc^3(x) + \csc(x)$$

Substitute and combine like terms:

$$f'(x) = \csc(x) - 2 \csc^3(x) - \sin(2x)$$

Final Answer:

$$\boxed{f'(x) = \csc(x) - 2 \csc^3(x) - \sin(2x)}$$