

Question 3

Let

$$f(x) = \sin(x) \cos(x) + \tan(x) \sec(x)$$

- (a) Find $f'(x)$.
- (b) Simplify the expression.

Question 3 - Solution

We are given:

$$f(x) = \sin(x) \cos(x) + \tan(x) \sec(x)$$

(a) Differentiate each term:

Term 1: $\sin(x) \cos(x)$

Using the product rule:

$$\frac{d}{dx}[\sin(x) \cos(x)] = \cos(x) \cos(x) + \sin(x)(-\sin(x)) = \cos^2(x) - \sin^2(x)$$

Term 2: $\tan(x) \sec(x)$

Using the product rule:

$$\frac{d}{dx}[\tan(x) \sec(x)] = \sec^2(x) \sec(x) + \tan(x) \sec(x) \tan(x) = \sec^3(x) + \sec(x) \tan^2(x)$$

(b) Combine and simplify:

$$f'(x) = \cos^2(x) - \sin^2(x) + \sec^3(x) + \sec(x) \tan^2(x)$$

Use trigonometric identities:

$$\cos^2(x) - \sin^2(x) = \cos(2x)$$

$$\tan^2(x) = \sec^2(x) - 1$$

$$\sec(x) \tan^2(x) = \sec^3(x) - \sec(x)$$

Substitute and combine like terms:

$$f'(x) = \cos(2x) + 2 \sec^3(x) - \sec(x)$$

Final Answer:

$$\boxed{f'(x) = \cos(2x) + 2 \sec^3(x) - \sec(x)}$$