

Question 2

Let

$$f(x) = x^2 \sec(x) + x \cot(x)$$

- (a) Compute the derivative $f'(x)$.
- (b) Simplify your result as much as possible.

Question 2 - Solution

We are given:

$$f(x) = x^2 \sec(x) + x \cot(x)$$

(a) Differentiate each term:

We apply the product rule to both terms:

First Term: $x^2 \sec(x)$

Let $u = x^2$, $v = \sec(x)$, then:

$$\frac{d}{dx}[x^2 \sec(x)] = \frac{d}{dx}[u \cdot v] = u'v + uv' = 2x \sec(x) + x^2 \sec(x) \tan(x)$$

Second Term: $x \cot(x)$

Let $u = x$, $v = \cot(x)$, then:

$$\frac{d}{dx}[x \cot(x)] = u'v + uv' = 1 \cdot \cot(x) + x(-\csc^2(x)) = \cot(x) - x \csc^2(x)$$

(b) Combine all terms:

$$f'(x) = 2x \sec(x) + x^2 \sec(x) \tan(x) + \cot(x) - x \csc^2(x)$$

Final Answer:

$$f'(x) = 2x \sec(x) + x^2 \sec(x) \tan(x) + \cot(x) - x \csc^2(x)$$