

Question 9

Let

$$f(x) = (x^2 + 1) \tan(x)$$

- (a) Differentiate $f(x)$ using the product rule.
- (b) Determine all x for which $f'(x)$ is undefined in the interval $(-\pi, \pi)$.

Question 9 - Solution

We are given:

$$f(x) = (x^2 + 1) \tan(x)$$

Let: $u(x) = x^2 + 1$ $v(x) = \tan(x)$

We'll use the product rule:

$$f'(x) = u'(x)v(x) + u(x)v'(x)$$

Step 1: Differentiate $u(x)$:

$$u'(x) = 2x$$

Step 2: Differentiate $v(x) = \tan(x)$:

$$v'(x) = \sec^2(x)$$

Now apply the product rule:

$$f'(x) = 2x \tan(x) + (x^2 + 1) \sec^2(x)$$

Final Answer:

$$f'(x) = 2x \tan(x) + (x^2 + 1) \sec^2(x)$$

(b) Where is $f'(x)$ undefined on $(-\pi, \pi)$?

$\tan(x)$ and $\sec^2(x)$ are undefined where $\cos(x) = 0$

$\cos(x) = 0$ at $x = -\frac{\pi}{2}, \frac{\pi}{2}$ in the given interval

So, $f'(x)$ is undefined at:

$$x = -\frac{\pi}{2}, \frac{\pi}{2}$$