

Question 8

Let

$$f(x) = \frac{e^x \cos x}{\ln(x)}$$

- (a) Compute $f'(x)$ using the quotient rule.
- (b) State the domain of $f'(x)$.

Question 8 - Solution

We are given:

$$f(x) = \frac{e^x \cos x}{\ln(x)}$$

Let: $u(x) = e^x \cos x$ - $v(x) = \ln(x)$

We'll use the **quotient rule**:

$$f'(x) = \frac{u'(x)v(x) - u(x)v'(x)}{[v(x)]^2}$$

Step 1: Differentiate $u(x) = e^x \cos x$ using the **product rule**:

$$u'(x) = e^x \cos x + e^x(-\sin x) = e^x(\cos x - \sin x)$$

Step 2: Differentiate $v(x) = \ln(x)$:

$$v'(x) = \frac{1}{x}$$

Now plug into the quotient rule:

$$f'(x) = \frac{e^x(\cos x - \sin x) \cdot \ln x - e^x \cos x \cdot \frac{1}{x}}{(\ln x)^2}$$

Factor e^x from the numerator:

$$f'(x) = \frac{e^x \left[(\cos x - \sin x) \ln x - \frac{\cos x}{x} \right]}{(\ln x)^2}$$

Final Answer:

$$f'(x) = \frac{e^x \left[(\cos x - \sin x) \ln x - \frac{\cos x}{x} \right]}{(\ln x)^2}$$

Domain of $f'(x)$:

$\ln x$ is defined only for $x > 0$

Also, $\ln x \neq 0 \Rightarrow x \neq 1$

And denominator has $\ln(x)^2$, so $x = 1$ is excluded

The term $\frac{\cos x}{x}$ is defined for $x > 0$

Domain:

$$(0, 1) \cup (1, \infty)$$