

**Question 7**

Let

$$f(x) = \frac{x \sin x}{x^2 + 1}$$

- (a) Compute  $f'(x)$  using the quotient rule.
- (b) Simplify the derivative fully.

**Question 7 - Solution**

We are given:

$$f(x) = \frac{x \sin x}{x^2 + 1}$$

Let:

$$u(x) = x \sin x, \quad v(x) = x^2 + 1$$

We apply the **quotient rule**:

$$f'(x) = \frac{u'(x)v(x) - u(x)v'(x)}{[v(x)]^2}$$

First, compute  $u'(x)$  using the product rule:

$$u'(x) = \sin x + x \cos x$$

Next, compute  $v'(x)$ :

$$v'(x) = 2x$$

Now apply the quotient rule:

$$f'(x) = \frac{(\sin x + x \cos x)(x^2 + 1) - x \sin x(2x)}{(x^2 + 1)^2}$$

Expand the numerator:

$$(\sin x)(x^2 + 1) + x \cos x(x^2 + 1) - 2x^2 \sin x$$

Distribute:

$$x^2 \sin x + \sin x + x^3 \cos x + x \cos x - 2x^2 \sin x$$

Combine like terms:

$$-x^2 \sin x + \sin x + x^3 \cos x + x \cos x$$

Factor where possible:

$$\sin x(1 - x^2) + x \cos x(x^2 + 1)$$

**Final Answer:**

$$f'(x) = \frac{\sin x(1 - x^2) + x \cos x(x^2 + 1)}{(x^2 + 1)^2}$$