

Question 5

Let

$$f(x) = x^2 e^x \cos x$$

- (a) Compute $f'(x)$ using appropriate differentiation rules.
- (b) Factor the result where possible.

Question 5 - Solution

We are given:

$$f(x) = x^2 e^x \cos x$$

This is a product of three functions: - $u(x) = x^2$ - $v(x) = e^x$ - $w(x) = \cos x$

Use the **product rule** for three functions:

$$f'(x) = u'(x)v(x)w(x) + u(x)v'(x)w(x) + u(x)v(x)w'(x)$$

Compute derivatives: - $u'(x) = 2x$ - $v'(x) = e^x$ - $w'(x) = -\sin x$

Now substitute:

$$f'(x) = 2xe^x \cos x + x^2 e^x \cos x - x^2 e^x \sin x$$

Factor where possible:

$$f'(x) = e^x (\cos x (2x + x^2) - x^2 \sin x)$$

Final Answer:

$$f'(x) = e^x [\cos x (x^2 + 2x) - x^2 \sin x]$$