

Question 3

Let

$$f(x) = \frac{\ln(x)}{x \cos(x)}$$

- (a) Differentiate $f(x)$ using the quotient and product rules.
- (b) Simplify your answer as much as possible.

Question 3 - Solution

We are given:

$$f(x) = \frac{\ln(x)}{x \cos(x)}$$

Let

$$u(x) = \ln(x), \quad v(x) = x \cos(x)$$

Then by the **quotient rule**:

$$f'(x) = \frac{v \cdot u' - u \cdot v'}{v^2}$$

Compute each derivative:

$$u'(x) = \frac{1}{x}$$

For $v(x) = x \cos(x)$, use the **product rule**:

$$v'(x) = \frac{d}{dx}[x \cos(x)] = \cos(x) - x \sin(x)$$

Now substitute into the quotient rule:

$$f'(x) = \frac{x \cos(x) \cdot \frac{1}{x} - \ln(x)(\cos(x) - x \sin(x))}{(x \cos(x))^2}$$

Simplify numerator:

$$f'(x) = \frac{\cos(x) - \ln(x) \cos(x) + \ln(x)x \sin(x)}{x^2 \cos^2(x)}$$

Final Answer:

$$f'(x) = \frac{\cos(x)(1 - \ln(x)) + \ln(x)x \sin(x)}{x^2 \cos^2(x)}$$