

Question 2

Let

$$f(x) = \frac{x^2 \sin(x)}{x + 1}$$

- (a) Differentiate $f(x)$ using the quotient rule.
- (b) Simplify your answer as much as possible.

Question 2 - Solution

We are given:

$$f(x) = \frac{x^2 \sin(x)}{x+1}$$

Let

$$u(x) = x^2 \sin(x), \quad v(x) = x+1$$

Then using the **quotient rule**:

$$f'(x) = \frac{v \cdot u' - u \cdot v'}{v^2}$$

We compute:

$$u'(x) = \frac{d}{dx}[x^2 \sin(x)] = 2x \sin(x) + x^2 \cos(x) \quad (\text{product rule})$$

$$v'(x) = \frac{d}{dx}[x+1] = 1$$

Now plug into the quotient rule:

$$f'(x) = \frac{(x+1)(2x \sin(x) + x^2 \cos(x)) - x^2 \sin(x) \cdot 1}{(x+1)^2}$$

Now expand and simplify the numerator:

$$= \frac{(x+1)(2x \sin(x) + x^2 \cos(x)) - x^2 \sin(x)}{(x+1)^2}$$

Distribute:

$$= \frac{2x(x+1) \sin(x) + x^2(x+1) \cos(x) - x^2 \sin(x)}{(x+1)^2}$$

Final simplified expression:

$f'(x) = \frac{2x(x+1) \sin(x) - x^2 \sin(x) + x^2(x+1) \cos(x)}{(x+1)^2}$
--