

Question 1

Let

$$f(x) = x^2 \cdot \ln(x)$$

- (a) Differentiate $f(x)$ using the product rule.
- (b) Find the equation of the tangent line to the graph of $f(x)$ at $x = 1$.

Question 1 - Solution

We are given:

$$f(x) = x^2 \cdot \ln(x)$$

Let:

$$u = x^2, \quad v = \ln(x) \Rightarrow u' = 2x, \quad v' = \frac{1}{x}$$

Apply the product rule:

$$f'(x) = u'v + uv' = 2x \ln(x) + x^2 \cdot \frac{1}{x} = 2x \ln(x) + x$$

Final derivative:

$$f'(x) = 2x \ln(x) + x$$

Now evaluate the tangent line at $x = 1$:

$$f(1) = 1^2 \cdot \ln(1) = 0$$

$$f'(1) = 2(1) \ln(1) + 1 = 0 + 1 = 1$$

Point on the curve: $(1, 0)$ Slope of tangent: 1

Use point-slope form:

$$y - 0 = 1(x - 1) \Rightarrow y = x - 1$$

Equation of the tangent line:

$$y = x - 1$$