

Question 10

Let

$$f(x) = \frac{\sin(x)}{1 + \cos(x)}$$

- (a) Differentiate $f(x)$ using the quotient rule.
- (b) Simplify the result as much as possible.

Question 10 - Solution

We are given:

$$f(x) = \frac{\sin(x)}{1 + \cos(x)}$$

Let:

$$u = \sin(x), \quad v = 1 + \cos(x)$$

Then:

$$f'(x) = \frac{v \cdot u' - u \cdot v'}{v^2}$$

Compute the derivatives:

$$u' = \cos(x), \quad v' = -\sin(x)$$

Apply the quotient rule:

$$f'(x) = \frac{(1 + \cos(x)) \cos(x) - \sin(x)(-\sin(x))}{(1 + \cos(x))^2}$$

Simplify numerator:

$$(1 + \cos(x)) \cos(x) + \sin^2(x) = \cos(x) + \cos^2(x) + \sin^2(x)$$

Use the identity $\sin^2(x) + \cos^2(x) = 1$:

$$= \cos(x) + 1$$

So:

$$f'(x) = \frac{\cos(x) + 1}{(1 + \cos(x))^2}$$

Cancel common factor:

$$f'(x) = \frac{1}{1 + \cos(x)}$$

Final Answer:

$f'(x) = \frac{1}{1 + \cos(x)}$
