

Question 8

Let

$$f(x) = \ln \left(\sqrt{x^2 + 3x + 2} \right)$$

- (a) Simplify the function before differentiating.
- (b) Find $f'(x)$.

Question 8 - Solution**(a) Simplify the Function:**

We start with:

$$f(x) = \ln\left(\sqrt{x^2 + 3x + 2}\right)$$

Since

$$\sqrt{x^2 + 3x + 2} = (x^2 + 3x + 2)^{1/2},$$

we can write:

$$f(x) = \ln\left((x^2 + 3x + 2)^{1/2}\right)$$

Using the logarithm rule

$$\ln(A^r) = r \ln(A),$$

we get:

$$f(x) = \frac{1}{2} \ln(x^2 + 3x + 2)$$

So the simplified function is:

$$f(x) = \frac{1}{2} \ln(x^2 + 3x + 2)$$

(b) Find $f'(x)$:

Now differentiate:

$$f(x) = \frac{1}{2} \ln(x^2 + 3x + 2)$$

Using

$$\frac{d}{dx} \ln(u) = \frac{u'}{u},$$

where

$$u = x^2 + 3x + 2$$

and

$$u' = 2x + 3,$$

we get:

$$f'(x) = \frac{1}{2} \cdot \frac{2x + 3}{x^2 + 3x + 2}$$

Therefore,

$$f'(x) = \frac{2x + 3}{2(x^2 + 3x + 2)}$$