

Question 7

Let

$$f(x) = \frac{e^x \sin x}{x^2 + 1}$$

- (a) Differentiate $f(x)$ using the quotient rule.
- (b) Evaluate $f'(0)$.

Question 7 - Solution

We are given:

$$f(x) = \frac{e^x \sin x}{x^2 + 1}$$

(a) Use the Quotient Rule:

The quotient rule is:

$$f(x) = \frac{u(x)}{v(x)} \Rightarrow f'(x) = \frac{u'v - uv'}{v^2}$$

Let:

$$u(x) = e^x \sin x, \quad v(x) = x^2 + 1$$

Differentiate $u(x)$ using the product rule:

$$u'(x) = \frac{d}{dx}[e^x \sin x] = e^x \sin x + e^x \cos x = e^x(\sin x + \cos x)$$

Differentiate $v(x)$:

$$v'(x) = \frac{d}{dx}[x^2 + 1] = 2x$$

Now apply the quotient rule:

$$f'(x) = \frac{e^x(\sin x + \cos x)(x^2 + 1) - e^x \sin x \cdot 2x}{(x^2 + 1)^2}$$

Factor out e^x :

$$f'(x) = \frac{e^x [(\sin x + \cos x)(x^2 + 1) - 2x \sin x]}{(x^2 + 1)^2}$$

(b) Evaluate $f'(0)$:

Substitute $x = 0$:

Numerator:

$$e^0 [(\sin 0 + \cos 0)(0^2 + 1) - 2(0) \sin 0] = 1 \cdot (0 + 1)(1) = 1$$

Denominator:

$$(0^2 + 1)^2 = 1^2 = 1$$

So:

$$f'(0) = \boxed{1}$$