

Question 6

Let

$$f(x) = \sqrt{x^2 + 4x + 5}$$

- (a) Differentiate $f(x)$ using the chain rule.
- (b) Find all values of x where the tangent line to $f(x)$ is horizontal.

Question 6 - Solution

We are given:

$$f(x) = \sqrt{x^2 + 4x + 5} = (x^2 + 4x + 5)^{1/2}$$

(a) Differentiate $f(x)$:

Using the chain rule:

$$f'(x) = \frac{1}{2}(x^2 + 4x + 5)^{-1/2} \cdot (2x + 4)$$

Simplify:

$$f'(x) = \frac{2x + 4}{2\sqrt{x^2 + 4x + 5}}$$

So,

$$f'(x) = \boxed{\frac{2x + 4}{2\sqrt{x^2 + 4x + 5}}}$$

(b) Horizontal Tangents:

The tangent is horizontal when $f'(x) = 0$. That occurs when the numerator is zero:

$$2x + 4 = 0 \Rightarrow x = -2$$

Check that the denominator is defined at $x = -2$:

$$\sqrt{(-2)^2 + 4(-2) + 5} = \sqrt{4 - 8 + 5} = \sqrt{1} = 1$$

So it's defined, and the tangent is horizontal at $x = -2$.

Conclusion:

- $f'(x) = \frac{2x + 4}{2\sqrt{x^2 + 4x + 5}}$
- Horizontal tangent at $x = \boxed{-2}$