

Question 4

Let

$$f(x) = \frac{\arctan x}{1 + x^2} + x^5 e^x.$$

- (a) Compute $f'(x)$.
- (b) Write the equation for critical numbers and approximate one in $(-5, -4)$ to six decimal places.

Question 4 - Solution**(a) Differentiate**

The quotient and product rules give

$$f'(x) = \frac{1 - 2x \arctan x}{(1 + x^2)^2} + e^x x^4 (x + 5).$$

(b) Critical numbers

The derivative exists for every real x , since $1 + x^2 > 0$.

Thus critical numbers solve $f'(x) = 0$. Bisection on $(-5, -4)$ gives

$$x \approx -4.995520.$$

The derivative has opposite signs at the ends of this interval, so continuity guarantees a root there.